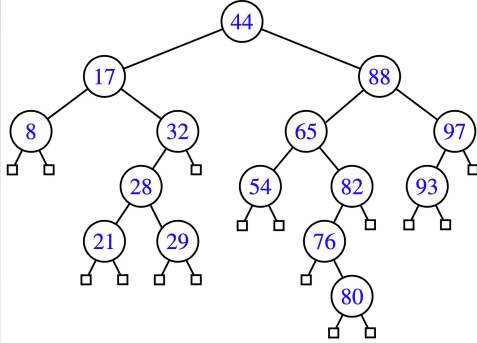
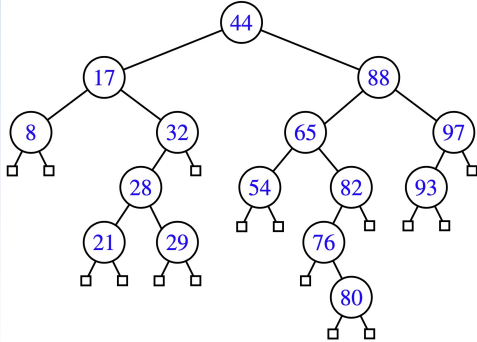


Visualizing BST Operation: Deletion

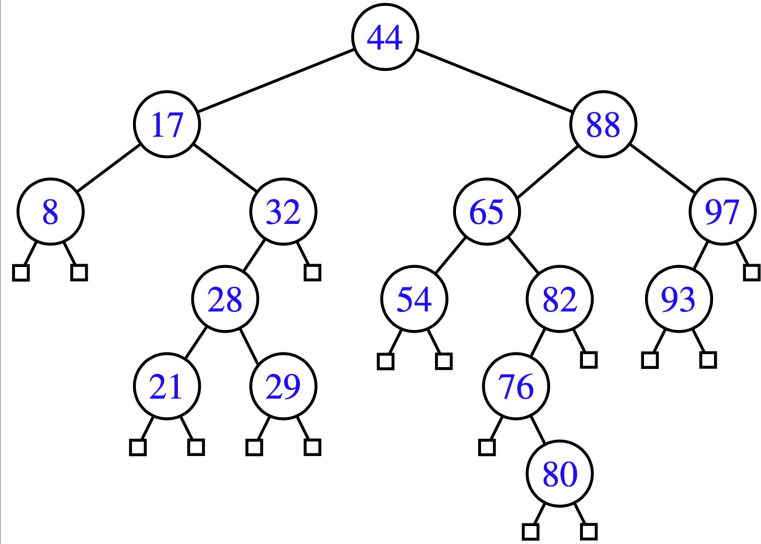
Case 1: Delete Entry with Key 31



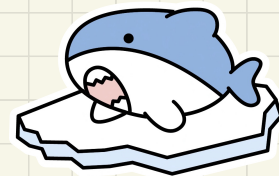
Case 2: Delete Entry with Key 80



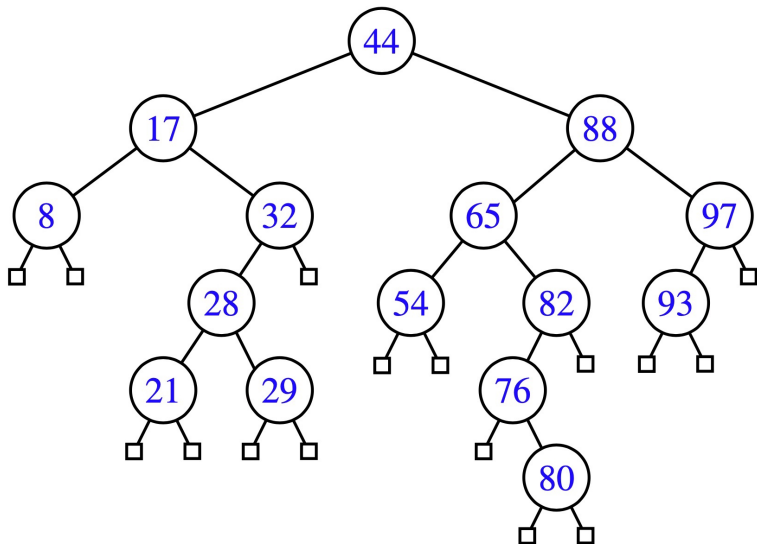
Case 3: Delete Entry with Key 32



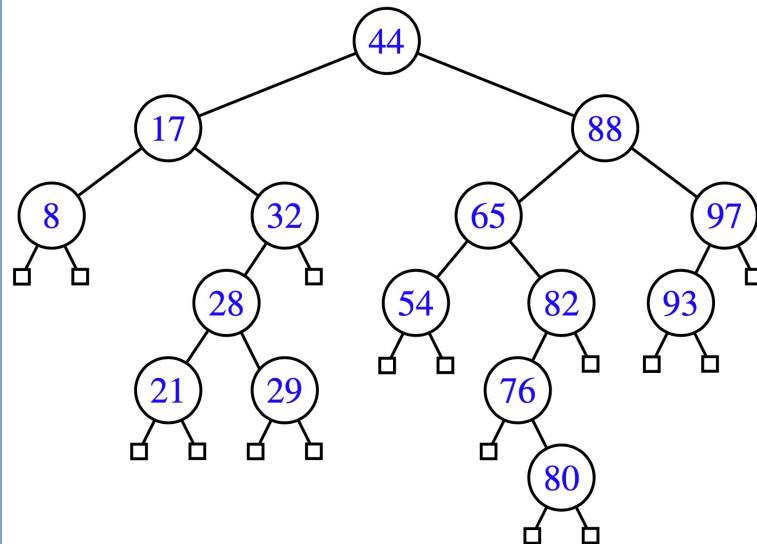
Visualizing BST Operation: Deletion



Case 4.1: Delete Entry with Key 17



Case 4.2: Delete Entry with Key 88



Top-Down Heap Construction

Problem: Build a heap out of N entries, supplied one at a time.

- Initialize an *empty heap* h .
- As each new entry $e = (k, v)$ is supplied, insert e into h .



Exercise: Build a *heap* out of the following 15 keys:

$\langle 16, 15, 4, 12, 6, 7, 23, 20, 25, 9, 11, 17, 5, 8, 14 \rangle$

Assumption: Key values supplied one at a time.

Bottom-Up Heap Construction

Problem: Build a heap out of N entries, supplied all at once.

- **Assume:** The resulting heap will be **completely filled** at all levels.
 $\Rightarrow N = 2^{h+1} - 1$ for some **height** $h \geq 1$ [$h = (\log(N + 1)) - 1$]
- Perform the following steps called **Bottom-Up Heap Construction** :

Step 1: Treat the first $\frac{N+1}{2^1}$ list entries as heap roots.

$\therefore \frac{N+1}{2^1}$ heaps with height 0 and size $2^1 - 1$ constructed.

Step 2: Treat the next $\frac{N+1}{2^2}$ list entries as heap roots.

◇ Each **root** sets two heaps from **Step 1** as its **LST** and **RST**.

◇ Perform **down-heap bubbling** to restore **HOP** if necessary.

$\therefore \frac{N+1}{2^2}$ heaps, each with height 1 and size $2^2 - 1$, constructed.

...

Step $h + 1$: Treat next $\frac{N+1}{2^{h+1}} = \frac{(2^{h+1}-1)+1}{2^{h+1}} = 1$ list entry as heap root.

◇ Each **root** sets two heaps from **Step h** as its **LST** and **RST**.

◇ Perform **down-heap bubbling** to restore **HOP** if necessary.

$\therefore \frac{N+1}{2^{h+1}} = 1$ heap, each with height h and size $2^{h+1} - 1$, constructed.

Exercise: Build a **heap** out of the following 15 keys:

<16, 15, 4, 12, 6, 7, 23, 20, 25, 9, 11, 17, 5, 8, 14>

Assumption: Key values supplied all at once.



```
graph TD; Root["(4,C)"] --- L1L["(5,A)"]; Root --- L1R["(6,Z)"]; L1L --- L2L1["(15,K)"]; L1L --- L2L2["(9,F)"]; L1R --- L2R1["(7,Q)"]; L1R --- L2R2["(20,B)"]; L2L1 --- L3L1["(16,X)"]; L2L1 --- L3L2["(25,J)"]; L2L2 --- L3L3["(14,E)"]; L2L2 --- L3L4["(12,H)"]; L2R1 --- L3R1["(11,S)"]; L2R1 --- L3R2["(13,W)"];
```

[illegible]